

### ✓ CHECK YOUR UNDERSTANDING

A babysitter is pushing a child on a swing. At the point where the swing reaches  $x$ , where would the corresponding point on a wave of this motion be located?

#### Solution

$x$  is the maximum deformation, which corresponds to the amplitude of the wave. The point on the wave would either be at the very top or the very bottom of the curve.



### PHET EXPLORATIONS

#### Masses and Springs

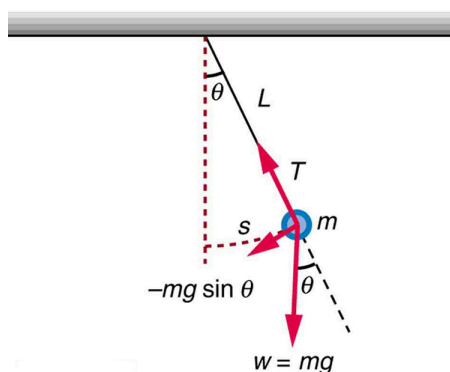
A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energy for each spring.

[Click to view content \(https://phet.colorado.edu/sims/mass-spring-lab/mass-spring-lab\\_en.html\)](https://phet.colorado.edu/sims/mass-spring-lab/mass-spring-lab_en.html)

Figure 16.13



## 16.4 The Simple Pendulum



**Figure 16.14** A simple pendulum has a small-diameter bob and a string that has a very small mass but is strong enough not to stretch appreciably. The linear displacement from equilibrium is  $s$ , the length of the arc. Also shown are the forces on the bob, which result in a net force of  $-mg \sin \theta$  toward the equilibrium position—that is, a restoring force.

Pendulums are in common usage. Some have crucial uses, such as in clocks; some are for fun, such as a child's swing; and some are just there, such as the sinker on a fishing line. For small displacements, a pendulum is a simple harmonic oscillator. A **simple pendulum** is defined to have an object that has a small mass, also known as the pendulum bob, which is suspended from a light wire or string, such as shown in [Figure 16.14](#). Exploring the simple pendulum a bit further, we can discover the conditions under which it performs simple harmonic motion, and we can derive an interesting expression for its period.

We begin by defining the displacement to be the arc length  $s$ . We see from [Figure 16.14](#) that the net force on the bob is tangent to the arc and equals  $-mg \sin \theta$ . (The weight  $mg$  has components  $mg \cos \theta$  along the string and  $mg \sin \theta$  tangent to the arc.) Tension in the string exactly cancels the component  $mg \cos \theta$  parallel to the string. This leaves a *net* restoring force back toward the equilibrium position at  $\theta = 0$ .

Now, if we can show that the restoring force is directly proportional to the displacement, then we have a simple harmonic oscillator. In trying to determine if we have a simple harmonic oscillator, we should note that for small angles (less than about  $15^\circ$ ),  $\sin \theta \approx \theta$  ( $\sin \theta$  and  $\theta$  differ by about 1% or less at smaller angles). Thus, for angles less than about  $15^\circ$ , the restoring force  $F$  is

$$F \approx -mg\theta.$$

16.23

The displacement  $s$  is directly proportional to  $\theta$ . When  $\theta$  is expressed in radians, the arc length in a circle is related to its radius (

$L$  in this instance) by:

$$s = L\theta, \quad 16.24$$

so that

$$\theta = \frac{s}{L}. \quad 16.25$$

For small angles, then, the expression for the restoring force is:

$$F \approx -\frac{mg}{L}s \quad 16.26$$

This expression is of the form:

$$F = -kx, \quad 16.27$$

where the force constant is given by  $k = mg/L$  and the displacement is given by  $x = s$ . For angles less than about  $15^\circ$ , the restoring force is directly proportional to the displacement, and the simple pendulum is a simple harmonic oscillator.

Using this equation, we can find the period of a pendulum for amplitudes less than about  $15^\circ$ . For the simple pendulum:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/L}}. \quad 16.28$$

Thus,

$$T = 2\pi\sqrt{\frac{L}{g}} \quad 16.29$$

for the period of a simple pendulum. This result is interesting because of its simplicity. The only things that affect the period of a simple pendulum are its length and the acceleration due to gravity. The period is completely independent of other factors, such as mass. As with simple harmonic oscillators, the period  $T$  for a pendulum is nearly independent of amplitude, especially if  $\theta$  is less than about  $15^\circ$ . Even simple pendulum clocks can be finely adjusted and accurate.

Note the dependence of  $T$  on  $g$ . If the length of a pendulum is precisely known, it can actually be used to measure the acceleration due to gravity. Consider the following example.



### EXAMPLE 16.5

#### Measuring Acceleration due to Gravity: The Period of a Pendulum

What is the acceleration due to gravity in a region where a simple pendulum having a length 75.000 cm has a period of 1.7357 s?

#### Strategy

We are asked to find  $g$  given the period  $T$  and the length  $L$  of a pendulum. We can solve  $T = 2\pi\sqrt{\frac{L}{g}}$  for  $g$ , assuming only that the angle of deflection is less than  $15^\circ$ .

#### Solution

1. Square  $T = 2\pi\sqrt{\frac{L}{g}}$  and solve for  $g$ :

$$g = 4\pi^2 \frac{L}{T^2}. \quad 16.30$$

2. Substitute known values into the new equation:

$$g = 4\pi^2 \frac{0.75000 \text{ m}}{(1.7357 \text{ s})^2}. \quad 16.31$$

3. Calculate to find  $g$ :

$$g = 9.8281 \text{ m/s}^2. \quad 16.32$$

### Discussion

This method for determining  $g$  can be very accurate. This is why length and period are given to five digits in this example. For the precision of the approximation  $\sin \theta \approx \theta$  to be better than the precision of the pendulum length and period, the maximum displacement angle should be kept below about  $0.5^\circ$ .

### Making Career Connections

Knowing  $g$  can be important in geological exploration; for example, a map of  $g$  over large geographical regions aids the study of plate tectonics and helps in the search for oil fields and large mineral deposits.

### Take Home Experiment: Determining $g$

Use a simple pendulum to determine the acceleration due to gravity  $g$  in your own locale. Cut a piece of a string or dental floss so that it is about 1 m long. Attach a small object of high density to the end of the string (for example, a metal nut or a car key). Starting at an angle of less than  $10^\circ$ , allow the pendulum to swing and measure the pendulum's period for 10 oscillations using a stopwatch. Calculate  $g$ . How accurate is this measurement? How might it be improved?

### ✓ CHECK YOUR UNDERSTANDING

An engineer builds two simple pendula. Both are suspended from small wires secured to the ceiling of a room. Each pendulum hovers 2 cm above the floor. Pendulum 1 has a bob with a mass of 10 kg. Pendulum 2 has a bob with a mass of 100 kg. Describe how the motion of the pendula will differ if the bobs are both displaced by  $12^\circ$ .

#### Solution

The movement of the pendula will not differ at all because the mass of the bob has no effect on the motion of a simple pendulum. The pendula are only affected by the period (which is related to the pendulum's length) and by the acceleration due to gravity.



### PHET EXPLORATIONS

#### Pendulum Lab

Play with one or two pendulums and discover how the period of a simple pendulum depends on the length of the string, the mass of the pendulum bob, and the amplitude of the swing. It's easy to measure the period using the photogate timer. You can vary friction and the strength of gravity. Use the pendulum to find the value of  $g$  on planet X. Notice the anharmonic behavior at large amplitude.

[Click to view content \(https://phet.colorado.edu/sims/pendulum-lab/pendulum-lab\\_en.html\)](https://phet.colorado.edu/sims/pendulum-lab/pendulum-lab_en.html)

Figure 16.15



## 16.5 Energy and the Simple Harmonic Oscillator

To study the energy of a simple harmonic oscillator, we first consider all the forms of energy it can have. We know from [Hooke's Law: Stress and Strain Revisited](#) that the energy stored in the deformation of a simple harmonic oscillator is a form of potential energy given by:

$$PE_{el} = \frac{1}{2}kx^2.$$

16.33

Because a simple harmonic oscillator has no dissipative forces, the other important form of energy is kinetic energy **KE**. Conservation of energy for these two forms is: